3236 Statistical Theory

4/4/23

Test of hypothesis:

Playing a game with a friend

flip a coin

if outcome is H $1.

outcome is T -$1

Assume that your friend provided The coin.

After 100 flips

1) Total gain ≃ 0

2) Total gain -50 probably not fair

3) Total gain -80 not fair.

6) Total gain -20 (??)

Shall you continue playing or not?

If p prob. of H.

H_0: $p \geq 0.5$

H_a: $p < 0.5$

H_a is the "working hypothesis",

H_0 is the hypothesis that brings more Trouble if rejected.

Use your data To:

a) not reject H_0

b) reject H_0

Test procedure.

$X_1 \ldots X_N$ $\quad N = 100$

coin flips

$H = 1$

$T = -1$

Sample space $= \{-1, 1\}^N = \Omega$

Partition

$$\Omega = S_0 \cup S_1$$

$S_0 \cap S_1 = \phi$

If outcome of N flips $\in S_1$

$\Rightarrow$ reject H_0.

S_1 critical region.

Choose a statistics

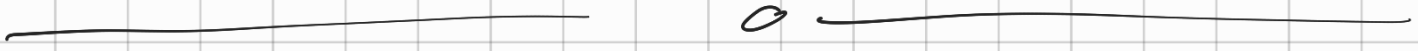
$$T = \sum_i X_i$$

$T < -C \Rightarrow$ reject H_0

$T \geq -C \Rightarrow$ do not reject H_0

T Test statistics

$\{-\infty, -c\}$ rejection region



If The coin is fair

you expect $T \approx 0$

if coin is unfair Toward me

Then prob. $T << 0$

How do I choose c ?

Assume That prop $H = p$

$$E(X_i) = p - (1-p) = 2p - 1 = \mu$$

$$V(X_i) = 4p(1-p) = \sigma^2$$

$$T \approx N(N\mu, N\sigma^2)$$

Errors:

Type I error: reject H_0 when it is True

Type II error: do not reject H_0 when it is false.

First choose a prob of Type I error I'm comfortable in accepting.

W. This all Tests That satisfy The above condition choose The one with minimal prob of Type 2 error.

I'm doing a Test on a parameter: p.

You Know That The coin was

selected at random from a bucket containing half coin with $p = 0.5$ and half $p = 0.4$.

$H_0 = p = 0.5$ $H_a : p = 0.4$

H_0 and H_a are simple.

If p is generic.

H_0 $p \geq 0.5$ $H_a : p < 0.5$

composite Hypotheses.

You may want To Test

$H_0 : p = 0.5$ $H_a : p \neq 0.5$

simple composite

Type I error:

H_0 is True $p \geq 0.5$

reject H_0 when $T < -c$

The greater is the True p

The smaller The prob of

rejecting H_0.

We want That for every $p \geq 0.5$ the prob of a Type I error is smaller Than The prob. we are willing To accept.

We need To check $p = 0.5$.

Calling α The prob of Type I error. We need To find $c > 0$ such That

$$P(T < -c \mid p = 0.5) < \alpha$$

α = significance level.